CENTRE OF MASS

Mass Moment : $\vec{M} = m\,\vec{r}$

CENTRE OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} \quad ; \quad \vec{r}_{cm}$$

$$= \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i} \quad \vec{r}_{cm} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i$$

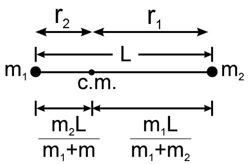
CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

$$x_{cm} = \frac{\int x\,dm}{\int dm},\ y_{cm} = \frac{\int y\,dm}{\int dm},\ z_{cm} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$ (mass of the body)

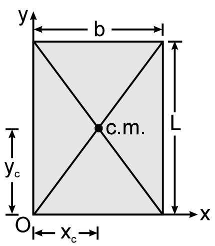
CENTRE OF MASS OF SOME COMMON SYSTEMS

⇒ A system of two point masses $m_1 r_1 = m_2 r_2$



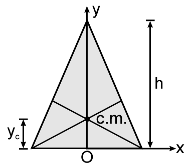
The centre of mass lies closer to the heavier mass.

⇒ Rectangular plate (By symmetry)



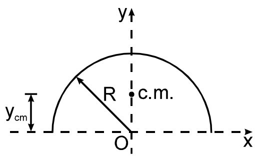
$x_c = \frac{b}{2}$ $\qquad$ $y_c = \frac{L}{2}$

⇒ A triangular plate (By qualitative argument)



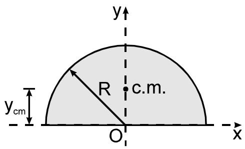
at the centroid : $y_c = \dfrac{h}{3}$

⇒ A semi-circular ring



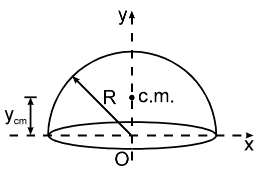
$y_c = \dfrac{2R}{\pi}$ $x_c = O$

⇒ A semi-circular disc



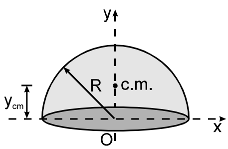
$y_c = \dfrac{4R}{3\pi}$ $x_c = O$

⇒ A hemispherical shell



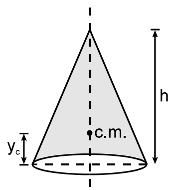
$y_c = \dfrac{R}{2}$ $x_c = O$

⇒ A solid hemisphere



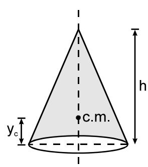
$y_c = \dfrac{3R}{8}$ $x_c = O$

⇒ A circular cone (solid)



$y_c = \dfrac{h}{4}$

⇒ A circular cone (hollow)



$y_c = \dfrac{h}{3}$

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM:

Velocity of centre of mass of system

$$\vec{v}_{cm} = \frac{m_1 \frac{\overrightarrow{dr_1}}{dt} + m_2 \frac{\overrightarrow{dr_2}}{dt} + m_3 \frac{\overrightarrow{dr_3}}{dt} .... + m_n \frac{\overrightarrow{dr_n}}{dt}}{M}$$

$$= \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 .... + m_n \vec{v}_n}{M}$$

$$\vec{P}_{System} = M \vec{v}_{cm}$$

Acceleration of centre of mass of system

$$\vec{a}_{cm} = \frac{m_1 \frac{\overrightarrow{dv_1}}{dt} + m_2 \frac{\overrightarrow{dv_2}}{dt} + m_3 \frac{\overrightarrow{dv_3}}{dt} .... + m_n \frac{\overrightarrow{dv_n}}{dt}}{M}$$

$$= \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 .... + m_n \vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net External Force} + \text{Net internal Force}}{M}$$

$$= \frac{\text{Net External Force}}{M}$$

$$\vec{F}_{ext} = M \vec{a}_{cm}$$

IMPULSE

Impulse of a force F action on a body is defined as :-

$$\vec{J} = \int_{t_i}^{t_f} F dt \qquad \vec{J} = \Delta \vec{P} \qquad \textit{(impulse - momentum theorem)}$$

Important points :

1. *Gravitational force and spring force are always non-impulsive.*
2. *An impulsive force can only be balanced by another impulsive force.*

COEFFICIENT OF RESTITUTION (e)

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r \, dt}{\int F_d \, dt}$$

$$= s \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

(a)	$e = 1$	$\Rightarrow$ *Impulse of Reformation = Impulse of Deformation*
		$\Rightarrow$ *Velocity of separation = Velocity of approach*
		$\Rightarrow$ *Kinetic Energy may be conserved*
		$\Rightarrow$ *Elastic collision.*
(b)	$e = 0$	$\Rightarrow$ *Impulse of Reformation = 0*
		$\Rightarrow$ *Velocity of separation = 0*
		$\Rightarrow$ *Kinetic Energy is not conserved*
		$\Rightarrow$ *Perfectly Inelastic collision.*
(c)	$0 < e < 1$	$\Rightarrow$ *Impulse of Reformation < Impulse of Deformation*
		$\Rightarrow$ *Velocity of separation < Velocity of approach*
		$\Rightarrow$ *Kinetic Energy is not conserved*
		$\Rightarrow$ *Inelastic collision.*

VARIABLE MASS SYSTEM :

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{rel}$ (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu|\vec{v}_{rel}|$.

Thrust Force ($\vec{F}_t$)

$$\vec{F}_t = \vec{v}_{rel}\left(\frac{dm}{dt}\right)$$

Rocket propulsion :

If gravity is ignored and initial velocity of the rocket u = 0;

$$v = v_r \ln\left(\frac{m_0}{m}\right).$$